TimeArcs: Visualizing Fluctuations in Dynamic Networks

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Abstract
In this paper we introduce TimeArcs, a novel visualization technique for representing dynamic relationships between entities in a network. Force-directed layouts provide a way to highlight related entities by positioning them near to each other. Entities are brought closer to each other (forming clusters) by forces applied on nodes and connections between nodes. In many application domains, relationships between entities are not temporally stable, which means that cluster structures and cluster memberships also may vary across time. Our approach merges multiple force-directed layouts at different time points into a single comprehensive visualization that provides a big picture overview of the most significant clusters within a user-defined period of time. TimeArcs also supports a range of interactive features, such as allowing users to drill-down in order to see details about a particular cluster. To highlight the benefits of this technique, we demonstrate its application to various datasets, including the IMDB co-star network, a dataset showing conflicting evidences within biomedical literature of protein interactions, and collocated popular phrases obtained from political blogs.

Categories and Subject Descriptors (according to ACM CCS): H.5.2 [Information Interfaces and Presentation]: User Interfaces—Graphical user interfaces

1. Introduction
Exploring relationships between entities collocated within an event or time period is a fundamental task for many visualization applications. Depending on the application domain, a relationship might occur when, for instance, two actors co-star in the same movie, two researchers co-author the same publication, or two proteins interact within a biological pathway. In many domains, relationships are time-dependent. For example, an actor may co-star with hundreds of other actors during his or her career. Moreover, the relationship between any pair of actors can be quite varied: some actors appear together only in a single movie, some work together consistently in multiple movies over a short period of time, and some are reunited after not having worked together for decades. The complexities associated with representing a large number of elements with dynamic connectivity make visualizing relationship networks challenging.

Bringing related entities close to each other allows a user to readily detect clusters within a large network. This provides a big picture view of entities and their temporal dynamics. For example, when analyzing newspaper articles grouping certain terms mentioned together in multiple news articles could be used to indicate or highlight a political event. Quickly identifying emerging patterns in local communities is a desired feature in many application domains, such as crime prevention where a sudden increase in phone calls between a group of people within an hour coupled with monetary transactions might be a sign of fraud taking place.

To address these challenges we developed TimeArcs, a novel visualization technique that makes it easy for a user to quickly identify patterns across time, and subsequently to analyze how those patterns might have formed and how they may evolve over time. Our technique utilizes constraints on a force-directed layout algorithm to automatically show patterns in text over time, as determined by a custom topic modeling algorithm or via features intrinsic to the original dataset. In this paper we introduce details about our interactive visualization technique for fluctuating dynamic networks, which: 1) presents the evolution of entities over time, 2) highlights temporal clusters of entities, and 3) supports various interactions that allow users to drill-down on a particular cluster or relationship of interest. Moreover, we provide demonstrations of the effectiveness of our technique through its application to three different real-world datasets, including: the collocated popular terms obtained from political blogs, the IMDB co-star network, and a dataset showing conflicting evidence within the biomedical literature of...
Figure 1: Visualizing collocated popular terms obtained from Wikinews in TimeArcs. Area graphs show how frequently the terms appear and are colored by term categorizations. Arcs highlight terms that appear together in the same articles. Interacting with the terms or arcs facilitates user exploration of temporal patterns within the topics that include those terms.

protein interactions. Fig. 1 shows an overview of TimeArcs applied to topics and terms extracted from Wikinews.

2. Related Work

2.1. Dynamic Network Visualization

With the increasing availability of temporal data, dynamic graph visualization is growing as an active research field with many applications in various domains. In a recent survey, Beck et al. [BBDW16] provide an overview of the growing number of techniques for representing the evolution of relationships between entities in readable, scalable, and effective diagrams. This survey presents a high-level categorization of different types of dynamic graph visualizations as animated node-link diagrams, timeline-based static charts, or hybrids of these. While the former has been a dominant method for dynamic visualizations, timeline-based techniques that provide a time-to-space mapping are becoming increasingly popular.

Greifich et al. [GBD09] propose a technique to visualize a weighted, dynamic compound digraph by drawing a sequence of node-link diagrams in a single view. Upward and downward edges are separated by using colored arcs. Horizontal alignment of nodes in the hierarchy at different time points are kept the same to facilitate comparison of the graphs in a sequence. This also represents a drawback of this technique: since the horizontal alignment of nodes is constrained by the hierarchical structure, nodes cannot be reordered to minimize edge crossings.

Parallel edge splatting [BVBB*11] takes a very different approach to visualizing dynamic graphs. In this technique, a sequence of narrow stripes are placed perpendicular to the horizontal timeline and hierarchically-organized vertices are arranged vertically within them. A relationship from A to B at time t is presented as a link from A at time t to B at time t + 1. Consequently, the dynamic graph looks similar to a parallel coordinates plot. Parallel edge splatting encounters the problem of visual clutter that occurs when drawing many lines onto a small portion of the screen space. To improve scalability on time axis, Beck et al. introduce Rapid Serial Visual Presentation (RSVP) [BBV*12], a hybrid approach mixing animated and timeline-based graph diagrams. A radial version [BBW12] of the Parallel edge splatting approach achieves shorter links than in the Cartesian counterpart. However, curved links in the radial technique seem to be harder to follow. Another radial approach, Radial Layered Matrix [VBSW13], produces less visual clutter by using radially distorted pixels instead of explicit link representations. An obvious drawback of this approach is that it can be difficult to identify trace connections between nodes.

Based on the Gestalt principles of closure, proximity, and similarity, van den Elzen et al. [vdEHBrW13] present node reordering strategies to enable users to find temporal properties such as trends, periodicity, and anomalies in a network. The paper also introduces strategies to reorder nodes vertically, such as minimizing edge length or reducing block overlap. However, these are NP-hard optimization problems [GJS74] and may not be appropriate in certain contexts.

Matrices can also be used to visualize the temporal changes in dynamic networks [BC02, MKF*15, YES10]. Adjacency matrices are particularly effective when visualizing dense graphs [HdF06] since they avoid edge-crossing
problem in node-link diagrams \cite{DMF15,GFC05,KEC06}. \textit{TimeMatrix} \cite{YES10} displays a small temporal bar chart within each cell of the matrix to show the changes of edge weights for the two corresponding vertices. Instead of bars, \textit{gestaltmatrix} \cite{BN11} uses \textit{gestaltlines}, intra-cell lines that encode different metrics using the angle and length. Individual time slices can be difficult to extract from matrix representations, but \textit{Matrix Cubes} \cite{BPF14} stacks adjacency matrices at each time step to form a space-time cube that can be decomposed into different 2D time slices or vertex slices. \textit{MultiPiles} \cite{BHRD15} presents the adjacency matrices (snapshots) side by side, and then similar consecutive snapshots are piled together to provide a more compressed view of a temporal network. A common drawback of all matrix representations is that paths between nodes are difficult to identify and trace.

2.2. Storyline Visualization

Storyline visualizations are inspired originally by Randall Munroe’s hand-drawn movie narrative charts\footnote{https://xkcd.com/657/}. Unique features of storyline visualizations, compared to other timeline visualization approaches, include each entity being represented as a line and that relationships between the entities being encoded according to the relative distances between the associated lines over time. Storyline visualizations have applications in different domains, such as tracing changes in family relationships in genealogical data \cite{KCH10}, understanding the evolution of community structures in dynamic social networks \cite{RTJ11}, and visualizing relationships between evolving topics in text streams \cite{CLT11,XWW13}.

Tanahashi and Ma \cite{TM12} propose a set of design considerations for generating storyline visualizations: reducing line crossings, maximizing the straightness and continuity of the lines, minimizing the wiggle distances to obtain a compact layout, and minimizing the empty space that may cause an unbalanced layout. A visualization based on these design principles can automatically generate a storyline layout, albeit taking considerable time to compute. The \textit{StoryFlow} \cite{LWW13} approach improves the speed of generating the storyline layout by using an efficient hybrid optimization approach. Furthermore, it embeds a contextual information hierarchy into the layout using closed contours surrounding the events in the background.

The \textit{TextFlow} visualization \cite{CLT11} enables the analysis of various evolution patterns that may emerge when examining multiple topics. Specifically, it focuses on the merging and splitting of relationships between evolving topics. Xu et al. \cite{XWW13} employed stacked graphs to display the time-varying “competitiveness” of topics on social media with a storyline style visualization. \textit{EvoRiver} \cite{SWL14} uses the same composite visual design, but separates threads of the topics into those which have a more negative or more positive sentiment. When there is a change in the “cooperation power” (from negative to positive or vice versa) the topic will switch to a different thread. Users can also select a time point and see relationships between different topics indicated by connected arcs, similar to the technique introduced in this paper.

In storyline visualizations, each entity in the visualization is represented as a line. This constraint makes storyline visualizations unsuitable for many application domains. While, for example, characters in a movie can only appear once in each scene at every time point (which is suitable for storyline visualizations), researchers often collaborate with different people to publish multiple papers in a year, or an actor may film with different crews concurrently (which is not suitable for storyline visualizations). As we show below, our \textit{TimeArcs} technique can highlight multiple relationships concurrently.

3. Design Decisions for the \textit{TimeArcs} Visualization

\textit{TimeArcs} is a timeline-based technique that facilitates the identification and exploration of temporal communities of network vertices and provides an overview of network dynamics over a given period of time. Given a set of entities and their relationships over time, \textit{TimeArcs} was designed with the following visualization goals (each of which was synthesized from the references provided immediately following each goal):

\textbf{G1. Display the evolution of entities as they change over time} \cite{BW08,FAHL11,DGWC10,XWW13}.

\textbf{G2. Highlight related entities by positioning them close to each other} \cite{CLT11,TM12}. This allows users to quickly identify temporal communities.

\textbf{G3. Reduce line/arc crossings that may lead to occlusion and visual clutter} \cite{TM12,DMAF15,LWW13}. Additionally, we also want to increase the legibility of text (i.e. entity labels) by minimizing the occlusion between texts and links.

Ahn et al. \cite{APS14} identify a task taxonomy for network evolution analysis across three dimensions: entity, property, and temporal feature. \textit{TimeArcs} specifically supports node/link level (G1) and group level (G2) entity analysis. Regarding the property dimension, \textit{TimeArcs} supports both structural properties (edge connectedness is used to organize entities as described in G2) and domain attributes (nodes/links are colored based on their categorizations). On the third dimension, \textit{TimeArcs} focuses on the temporal features of individual events. More specifically, our work aims to make it easy to discover at what point in time an entity, relationship, or group activity appears or disappears.

To satisfy the design criteria introduced above, we made the following design decisions:

\textbf{D1. The time axis is aligned horizontally from left to right.}
This design is widely used when visualizing time series data [HHWN02, Wat05, BW08].

D2. Each entity is represented as a straight line. Previous research has indicated that minimizing the crossings between entity representations is the most important metric to reduce visual clutter [LWW+13, Pur97]. It is easier to trace a straight line (to visualize the temporal relationships associated to an entity) than to trace a curve [MD12]. Moreover, a CloudLines style visualization [KBK11, LYK+12] can be overlaid to highlight the evolution of entities over time (design goal G1).

D3. Arcs are used to connect related entities. The forces applied on the arcs bring connected entities closer together on the vertical axis (design goal G2). Force-directed layouts are very useful in highlighting cluster structure without requiring the use of additional clustering algorithms. Furthermore, by bringing connected entities together we reduce the crossings between arcs of different temporal clusters (design goal G3).

Overall, TimeArcs can be considered a hybrid visualization that arranges CloudLines vertically in order to highlight the evolution of entities over time. The CloudLines are pulled closer together if these entities are connected at some time points using a force directed layout. Finally, arc diagrams are used to connect related entities at each time point.

Additional optimization strategies augment our design choices and generate visual output that is both more aesthetically pleasing and more legible. In TimeArcs, entities that appear closer together vertically are considered to be more related than entities at a distance from each other. The relatedness between entities is defined by: (1) the total number of connections at different time points, such as those that are between terms mentioned together multiple times; and (2) the weight (and strength) of the connections at particular points, such as those that are between terms mentioned together many times on a particular day (consequently connected by thicker and stronger arcs). We apply these factors onto the force-directed layouts to maximize the neighborhood of more related entities.

Since each entity in TimeArcs is represented as a straight line, an entity label can appear anywhere along this line. For example, an entity label can appear where the entity is frequently mentioned or where the entity is highly connected. After selecting the intended location for an entity label, we check if there are any self-occlusions between texts and arcs of the same entity. If there are, we continue moving the label to the left until we find a position without self-occlusions. Notice that this strategy removes any self-occlusions of individual entities but does not guarantee the removal of all occlusions in the graph. To guarantee the removal of all occlusions, we can move all of the text labels to the left of the layout. An entity label can be drawn repeatedly at different points along the time axis in order to reduce tracing time.

4. Computing the TimeArcs Visualization

In this section we describe the primary components of the TimeArcs visualization (also depicted in Fig. 2). These include the main components for computing the visual elements that represent the temporal evolution and relationships of ranked entities as well as the interactive elements that facilitate different ways of filtering by time or search terms:

- Computing the evolution of entities: This step counts the frequency of occurrences of entities at each time point across the entire temporal range and subsequently ranks them (Section 4.1).
- Computing the relationship of ranked entities: This step determines the relationship between entities. Two entities are considered to be related if they are collocated within the same contexts (Section 4.2).
- Selecting highly connected entities: This step identifies important entities among the highly ranked entities in the network (i.e., nodes that have high degree of centrality) and highlights them in the visualization. (Section 4.3).
- Filtering entities: This step facilitates user-driven filtering to interactively explore details of the data. TimeArcs supports multiple ways to filter entities, such as filtering by time or by a search term. Entity ranking and entity relationships are recomputed based on filtering conditions (Section 4.4).

4.1. Computing the Evolution of Entities

Input entities are available from input files, such as names of authors in publications or actors in movies. However, in other cases we need to preprocess the data in order to generate these entities, which could represent frequent terms or phrases extracted from text documents, blogs, or news articles. In such cases, we perform named-entity recognition on the text documents, which allows us to identify names of people, places, and organizations first and calculate their frequencies and co-occurrence afterwards.

Fig. 3 illustrates two typical examples of entity evolution.
visualizations: stacked graphs and small multiples. The data for these graphs were retrieved from political blogs in the 10-year period from 2005 to 2015. The top 50 terms that appeared in these blogs are highlighted by category: green for person, red for location, blue for organization, and yellow for miscellaneous. Primary benefits of using a stacked graph include its compactness and its ability to provide a comprehensive overview. Small multiples make it easier to trace and compare the evolutions of different terms over time (design goal G1).

Figure 3: Popular techniques to visualize entity evolutions: (a) stacked graph and (b) small multiples. Here, entities are terms extracted from political blogs and color-coded by category: green for person, red for location, blue for organization, and yellow for miscellaneous data. The two above graphs are implemented in D3.js.

In addition to showing the frequency data of entities over time, we propose the use of time series features to discover terms associated with events within the time series, such as sharp increases or drops [SJA*06, DW13], the sudden increase followed by a sudden drop [BAP*05], and serial periodicities [CK98]. In particular, we define a sudden attention measure for entities, referring to a sharp increase in frequency.

Let $F_1, F_2, \ldots, F_n$ be the frequency of an entity at $n$ different time points. Instead of ranking an entity based on its raw time series $(F_1, F_2, \ldots, F_n)$, we derive the sudden attention series $(A_1, A_2, \ldots, A_n): A_i = \frac{(F_i+1)}{(F_{i-1}+1)}$. For example, the frequency of the term “Obama” at time $t = 99$ (or $F_{t-1} = 99$) and at time $t = 199$ ($F_t = 199$). Then the attention of term “Ebola” at time $t$ is $A_t = \frac{(F_t+1)}{(F_{t-1}+1)} = 11$. Therefore at time $t$, the term “Ebola” is considered much more significant than the term “Obama” even though the frequency of “Ebola” at time $t$ is relatively small compared to that of “Obama.” This measure aims to detect entities which suddenly draw a lot of attention (and are usually connected with a particular event in the time series) rather than entities which are more consistently popular.

4.2. Computing the Relationships between Entities

This step computes the relationships between pairs of entities. In applications where the set of entities is large, such as the number of actors in IMDB database or the number of terms/phrases extracted from political blogs (our algorithm extracted 418,641 terms from 90,811 blogs spanning 10 years), computing relationships between all pairs is computationally expensive. We therefore rank the input entities based on their frequency or sudden attention score and only compute the relationships between highly ranked entities, for example, only the 1,000 top-ranked entities.

The relationships between entities are defined differently in various applications. In researcher collaboration networks, two researchers are related if they are co-authors of the same papers; while in a “money trail” inspection, two people are related if they communicate by phone or email or if they transfer money. The strength of a relationship is computed based on the number of collocations of two entities at a particular time point and is encoded in our visualization by the thickness of the link connecting the two entities.

4.3. Selecting Highly Connected Entities

Among highly ranked entities, we further identify and select the most highly connected entities. In other words, we want to include nodes with a high degree of centrality in the network. In social networks, nodes with a high degree of centrality represent the most influential people. Fig. 4 shows an example of most influential authors in the IEEE VIS conferences over the last five years. In particular, each graph is a snapshot of collaboration between these researchers in one year. We keep the nodes in the same positions and fade out unconnected nodes to help the viewer see the differences between these five snapshots. However, when the number of nodes and/or the number of snapshots increase, visualizing the dynamics of the network becomes difficult.

Using TimeArcs, we combine the five force-directed layouts into one. Besides the forces applied on links to pull connected entities together, we add two more kinds of forces into the layout: (1) Pull vertices representing the same entity at different time points to the same horizontal line, thus maintaining the mental association a user would create between line and entity (design choice D2); (2) Pull and align
vertices onto the middle vertical line to resolve any inconsistency between different clusters at multiple time steps. Finally, vertices are pinned to their corresponding horizontal coordinate on the time axis (to ensure design choice D1). Fig. 5 shows TimeArcs applied to the same data as seen in Fig. 4.

Fig. 5: The TimeArcs visualization applied to the IEEE VIS publication co-authorship network of the top 50 researchers from 2010 to 2014 (i.e., the same data in Fig. 4).

4.4. Filtering Entities

TimeArcs additionally supports multiple ways to filter data, including: (1) filtering connections by strength, (2) filtering by a time interval, and (3) focusing on a specified entity. As depicted in the TimeArcs schema (Fig. 2), once users apply a filtering condition both the ranking of entities and the entity relationships need to be recomputed. For example, when users input a new search term using a dataset of news items, term frequencies and their co-occurrences are recomputed based on the articles containing that search term. When users search for the collaborations of researchers within a different range of years (using the co-authorship network), the degree of centrality of the vertices in the network may change completely and thus need to be recomputed.

Fig. 6 shows an example of the collaboration networks of “Munzner, T.” from 1995 to 2014. In this visualization, we have ordered entities by the time when they are first connected to the search entities (along with other constraints in TimeArcs layout). This ensures that arcs appearing first have a smaller distance to the focused entity (“Munzner, T.” in this case) than the ones appearing after to avoid crossings (design goal G3). Thicker arcs connect researchers having multiple publications with “Munzner, T.” in a single year. Notice that “Tory, M.” (in the highlighted box) had multiple publications with “Munzner, T.” in 2003, 2007, 2010, and 2013.

Fig. 6: Visualizing collaboration networks for “Munzner, T.” over the past 20 years (green for the InfoVis conference, red for VAST, and blue for SciVis).

5. Applications

To demonstrate the usefulness and effectiveness of the TimeArcs visualization, we describe its application to three different datasets: one containing blog postings about political events, the IMDB co-star database, and a biomedical database providing evidences in the literature of protein interactions.

5.1. Exploring Topics and Events in Political Blogs

We collected 90,811 political blog posts over a ten-year period from 2005 to 2015 from seven different sources, including AMERICAblog, Huffington Post, and ProPublica. We
then ran text analyses on these blogs and generated terms that were classified into four different categories. These terms were then input into TimeArcs. We first computed the sudden attention measure (see Section 4.1) for each term and then computed the relationships between the top 1,000 terms. We filtered relationships of strength at least 15 (i.e., terms that were mentioned together in at least 15 blogs in one month). Finally, the top 100 terms with a high degree of centrality were plotted in the layout depicted in Fig. 7. This layout provides an overview of major political events in the past 10 years in one display.

5.2. Finding Patterns in the IMDB Co-Star Network

The data is available on IMDB website‡. We went through 9,963 movies rated 8 (out of 10) stars or higher from 1980 to 2014 across three genres: comedy, action, and drama. In total, our dataset contained 66,182 actors. Fig. 8 shows TimeArcs for the top 200 actors. In particular, the arcs connect co-actors in the same movies. In this use case, we color the arcs by movie genres: green for comedy, red for action, blue for drama. TimeArcs helps viewers to quickly identify temporal communities of actors. Each horizontal line represents one actor and connects his or her first through last appearances in highly rated movies. This helps to highlight actors with long careers and many good movies, such as, for example, the voice actor Michael Bell (at the red arrow in Fig 8). The horizontal lines can be replaced by CloudLines-style graphs on demand. By brushing any actors’ name, we can immediately visualize his or her co-star network to see how it changes over time.

Continuously repeated cliques of actors (as in the highlighted boxes A, B, and C of Fig. 8) usually indicate that they have appeared together in multiple seasons of a television series. For example, Box B shows “The Chaser Election Specials”, an Australian comedy TV series which appeared in 2001, 2004, 2007, and 2010. Box A highlights Marin Mandir’s movies, such as “Police, Follow that Car” (2001) and “Facebook dvojnik” (2012). Marin Mandir also acts in his own movies. Box C contains “ReBoot”, an action-adventure television series that originally aired from 1994 to 2001.

Though by default we initially limit our visualization to the top 200 actors, we can easily add more entities into the visualization, similar to the way items are added into a spreadsheet. This avoids text occlusion that can occur in standard force-directed layouts. Simple mouse scrolling can help users to navigate through the list easily, an advantage of laying entities vertically.

5.3. Evidence in Biological Pathway Literature

In this case study, we explore evidences from the biomedical literature describing protein interactions, retrieved from the Pathway Commons database§. The data contains the publication information (such as publication year, author, and textual evidence) of interactions between pairs of proteins, as well as their specific interaction types. Fig. 9 shows new discoveries in protein interaction networks from 2002 to 2013. An arc connects two proteins at the times when the interaction was jointly described in a publication together. The colors encode interaction types: green for adds_modification, red for removes_modification, blue for translocation, and

‡ http://www.imdb.com/interfaces
§ http://www.pathwaycommons.org/
Visualizing the IMDB co-star network of the top 200 actors from highly rated movies from 1980 to 2014. Arcs connect co-stars and are colored green for comedy, red for action, and blue for drama. Boxes A, B, and C highlight actors who appear together in multiple seasons of a series.

Figure 9: Visualizing the publication of new discoveries in protein interaction networks from 2002 to 2013. The colors encode different types of biochemical interactions.

When there are multiple arcs connecting two proteins, it falls into one of the two circumstances. If they have the same color, these arcs indicate that there are supporting evidences in different publications which confirm the interaction between two elements. On the other hand, if they have the different colors, the more recent appearance provides either more detailed knowledge about the interaction or shows a conflict between different articles regarding the way in which these proteins interact.

Figure 10: TimeArcs visualization for interactions around PCAF protein. (1), (2), and (3) in the figure are supporting evidences in literature of “PCAF binds MAML”.

In Fig. 10, TimeArcs visualizes interactions between PCAF protein complex and other biomolecules. In particular, above the PCAF timeline we can see there is new evidence from 2013 that supports the interaction “PCAF binds p300 and KAT3A”, which was first discovered in 2011. Similarly under PCAF timeline, there are three evidences supporting “PCAF binds MAML” in 2008, 2011, and 2013. On the other hand, in Fig. 11 TimeArcs depicts interactions between the OPSD protein and the K+ protein. Here we can see that the 2003 and 2012 publications are in conflict. Contradictorily, OPSD and K+ appear to both positively and negatively regulate each other.
The main contribution of the paper is our novel use of force-directed layouts as primary way to group related entities and minimize arc crossings, it thus shares some common features with it. (1) Gravity of the layout and repellers between vertices ensure that important vertices (having a high-degree of centrality) end up at the vertical center of the TimeArcs layout, while vertices with a low-degree centrality end up toward the top or bottom of the TimeArcs visualization. For example in Fig. 7, the term “Sarah Palin” is located in the center of both layouts while “Boston marathon” and “Dzhokhar Tsarnaev” are isolated and move away from the both centers. (2) TimeArcs is more suitable for sparser and fluctuating dynamic networks (which have temporal clusters changing over time). For denser graphs, force-directed layouts become “hairballs”. In these cases, users can use sliders to interactively filter out the weaker relationships between entities. To show all relationships between entities in dense dynamic networks, matrix representations (such as Matrix Cubes [BPF14] or MultPiles [BHRD*15]) are probably more appropriate.

References


