A Ray Tracer to Visualize Higher Dimensional Julia Sets

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Abstract

Artists, scientists and mathematicians have been collaborating on a variety of projects at the Electronic Visualization Laboratory for over thirty years. In 1989, a ray tracer was created to visualize higher dimensional Julia Sets, involving and contributing to advances in all three fields.

1. Background

The creation of the ray tracer to visualize higher dimensional Julia Sets described here took place in a research environment called the Electronic Visualization Laboratory (EVL). EVL is a shared facility of the School of Art and Design and the Department of Computer Science at the University of Illinois at Chicago. In operation for over 30 years, EVL has two directors, 12 associated faculty and staff, and 50 graduate students. About one-third of the students are pursuing a Master of Fine Arts (MFA) in the School of Art and Design and two-thirds are pursuing either a Master of Science or a Ph.D. in computer science. To our knowledge, this is the longest living program that offers an MFA which is a formal collaboration of art and computer science. EVL delivers art intelligence to science, and science and technology to artists. It systematically teaches the artists the technology and less systematically teaches the art to the computer science students. The artists' experience is central to the success of EVL. EVL creates and expands new media and supplies art and science content for those media.

The economic support of the EVL is based on working with scientists and mathematicians to deliver new visualization techniques and technologies to support scientific and mathematical investigation. The research outlined in this paper would not have been possible without EVL and the expertise of both the computer scientists and artists.

There has been considerable interest in the visualization of fractals. Their infinite detail combined with repeating forms have intrigued mathematicians and artists and broadly appeal to the general public. The most popular images have been of two-dimensional Mandelbrot and Julia Sets.

2. Methodology

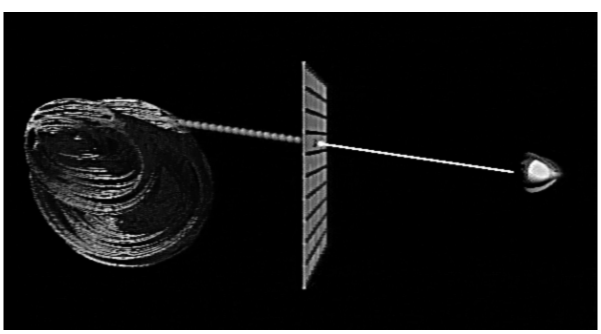
The Julia Set in the complex plane is defined as: a polynomial of the form $f(z) = z^2 + (a + bi)$ where z is a complex variable and (a+bi) is a fixed complex number. For each z in the complex plane one considers the iterates z, f(z), f(f(z)), ... If these iterates approach infinity, then the point z is not in the Julia Set associated with f(z). Julia Set J(f) is defined to be the set of points z that do not go to infinity, but that have points close to them going to infinity.

One method of computing these Julia Sets flows directly from the definition. The program takes every pixel and repeats the computation $Z = Z^2 + K$. If the magnitude of the point Z does not become very large in several hundred iterations it is colored as being contained in the Julia Set. This actually produces a filled-in Julia Set because it marks all points that do not escape to infinity, rather then coloring only the points that are close to those that go to infinity.

We were interested in visualizing Julia Sets in more than two dimensions. One can define the Julia Set for $Z = Z^2 + K$ in a four-dimensional space called the quaternions. Quaternions are a four-dimensional extension of complex numbers created by Sir William Hamilton in the 19th Century.

The Julia Sets can be computed by a method analogous to the two-dimensional case by taking every point in a three-dimensional space and iterate $Z = Z^2 + K$ several hundred times to see if it escapes. One major problem with this approach is that high-resolution images can take a very long time to compute. For a 512 x 512 two-dimensional Julia Set there are 250,000 pixels, but for a 512 x 512 x 512 image there are 13 million voxels. If it takes a half hour to compute the two-dimensional Julia Set it would take 10.5 days to compute a three-dimensional subset of the four-dimensional Julia Set.

A second major issue involves the way lighting is computed in three-dimensional computer graphics. To calculate smooth shading, the angle between a line from the light source to the surface of the object and the normal to the surface of the object is needed. Since fractals are infinitely wrinkled they do not have normals. In 1989 John Hart generalized an equation from complex numbers that gives a distance estimate between a point and the surface of a Julia Set in the quaternions [1]. By using this formula one could skip the vast majority of the voxels in the three-dimensional space and compute only the voxels near the quaternion Julia Set. In addition this distance estimation can be used to calculate an approximate normal so that lighting can be calculated. This work enabled the creation of ray traced quaternion Julia Set animations, and also contributed to computer science by inventing a new rendering technique.



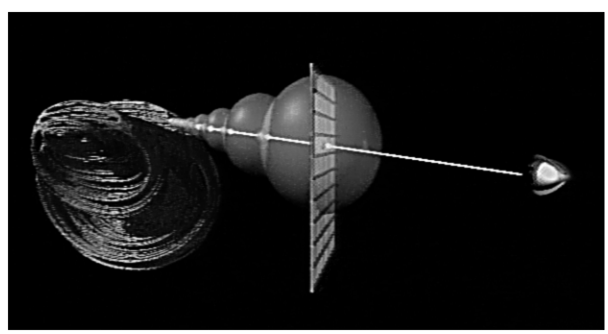


Figure 1 (left): There is no known equation for the intersection of a line and a fractal. To ray trace a fractal, one could step along the ray in small increments and iterate the function to see if it escapes. This method is very inefficient and requires long computation times.

Figure 2 (right): Using an estimate of the distance to the fractal, we can skip to the far side of the distance estimate, take another distance estimate, and converge toward the fractal.

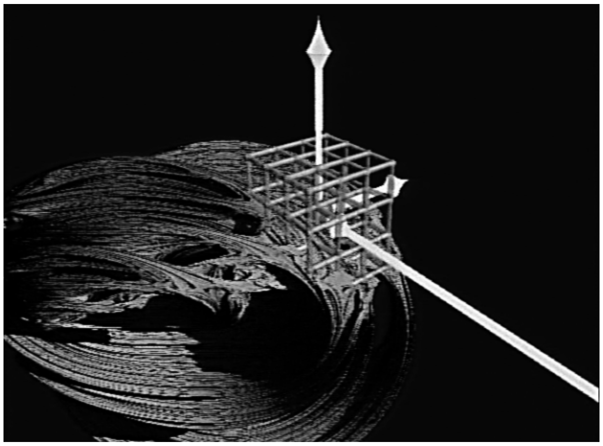
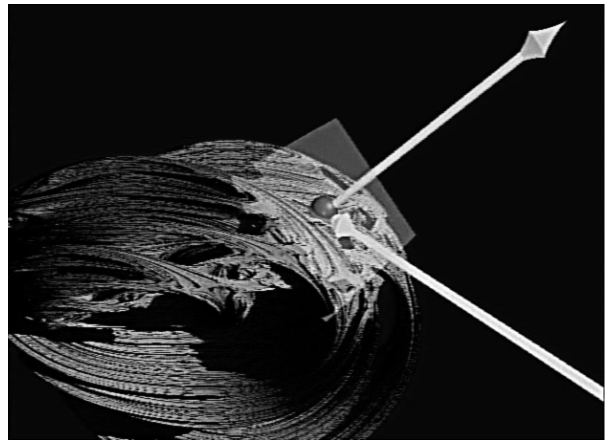
 

Figure 3 (left): A normal to the surface can be approximated by estimating the distance to the fractal from a regular lattice of points centered on the point of intersection.

Figure 4 (right): From the normal vector a reflection plane can be specified to bounce the ray for lighting calculations.

The equation for the distance estimate between a point and the quaternion Julia Set appeared to work well. The three-dimensional forms generated were the same as those generated by less efficient means. However there was no proof that this estimations technique was correct in the quaternions. In 1996, as part of her Ph.D. dissertation, Yumei Dang proved that the distance estimation formula was correct [2]. She also developed a distributed computational technique that could efficiently use a larger array of supercomputers to calculate the images, thereby contributing to the field of computer science.

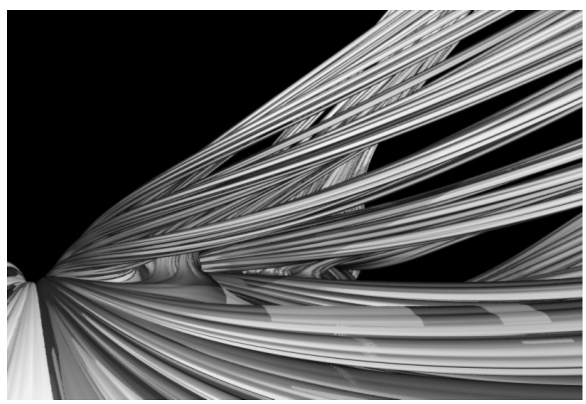
 

Fig 5 and 6: Animation images from "A Volume of Two Dimensional Julia Sets"

In 1990, Dan Sandin, Lou Kauffman and John Hart created an animation called "A Volume of Two Dimensional Julia Sets." This one minute ray traced animation premiered at the electronic theater as part of the SIGGRAPH 1990 conference. This three-dimensional fractal was created by stacking two-dimensional Julia Sets on top of each other. The distance estimation developed by John Hart could also be applied to this figure.

Computation times for video resolution (640 x 480 x 30fps) on the AT&T pixel machine took between 5 and 30 minutes. The AT&T pixel machine was an advanced graphics supercomputer (for the 1980's). A second animation "A Diamond of Quaternion Julia Sets" was commissioned for Nippon Telephone & Telegraph's 50th birthday party to feature their new Super High-Definition (SHD) video systems (2048 x 2048 x 60fps). This work was based on software developed by Yumei Dang and Lou Kauffman and was computed on an array of the SGI Onyx graphics supercomputers. Computation times ranged from five minutes to 30 minutes. I am currently working on a new longer SHD animation based on quaternion Julia Sets. Computation times between 5 and 30 minutes running on multiple Linux clusters are expected.

3. Conclusion

These collaborations have resulted in contributions to art, mathematics and computer science. For example, the collaboration related to the visualization of quaternionic Julia Sets resulted in a computer animation "A Volume of Two Dimensional Julia Sets" which has been exhibited at numerous art museums and video festivals. This research also resulted in the development of a new computer graphics rendering method by John Hart which was presented at SIGGRAPH 89, and the development in 1995 of the distributed method utilizing many supercomputers which was shown during Supercomputing 95. In 1996, Yumei Dang and Lou Kauffman proved that the distance estimations technique empirically developed by John Hart in 1989 was mathematically correct, resulting in new theorems. This mathematical research is collected in a recent book and CDROM [3]. In addition to the work presented here, Lou and I have collaborated on visualizations of knots, four-dimensional topologies, dynamical systems and cellular automata. These investigations have resulted in video tapes and interactive virtual reality installations in both art and science museums. We will continue to collaborate on a range of visualizations including fractals, knots, physics and everything.

References

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[2] Dang, Y. and Kauffman, L.H. (1997), Hypercomplex Fractal Distance Estimation, in M. M. Novak and T.G. Dewey, eds., Fractal Frontiers, World Scientific, 117-130.

[3] Yumei Dang, Louis H. Kauffman, Daniel J. Sandin, Hypercomplex Iterations: Distance Estimation and Higher Dimensional Fractals, River Edge, NJ: World Scientific, c 2002. (Series: K & E series on knots and everything; v. 17) QA297.8.D25 2002 <http://www.evl.uic.edu/hypercomplex/>